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CONDITIONS OF DYNAMIC SIMILARITY IN FLUIDIZED BEDS

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Based on the nondimensionalization of Anderson–Jackson equations, the conditions for the total similarity of hydrodynamic processes in fluidized beds are obtained. By introducing the excess rate of filtration $u - u_t^*$, which represents the natural scale of the particle velocity, account is taken of the most important specific feature of a fluidized system, i.e., the existence of large-scale circulation motions of the solid phase.

The revealing of similarity conditions for transfer processes is a very important problem from the viewpoint of both the formulation of scale-transition rules and the development of engineering methods for calculating apparatuses with disperse systems.

There are a number of publications known in the literature in which systems of nondimensional parameters that describe the similarity of transfer processes in fluidized beds are obtained. Reviews of these works are presented in [1, 2].

As of today, the following system should be recognized as the most justified:

$$\mathrm{Fr}_D, \ \frac{\rho_s}{\rho_f}, \ \bar{J}_s, \ \frac{d}{D}, \ \mathrm{Re}, \tag{1}$$

This system is obtained by means of nondimensionalization of mass-balance equations and by the phase pulses suggested by Anderson and Jackson [3]. The nondimensional numbers (1) give five independent equations

$$(\mathrm{Fr}_D)_1 = (\mathrm{Fr}_D)_2, \ \left(\frac{\rho_s}{\rho_f}\right)_1 = \left(\frac{\rho_s}{\rho_f}\right)_2, \ (\bar{J}_s)_1 = (\bar{J}_s)_2,$$
$$\left(\frac{d}{D}\right)_1 = \left(\frac{d}{D}\right)_2, \ \mathrm{Re}_1 = \mathrm{Re}_2 \tag{2}$$

for determining five parameters of a laboratory bench that models an actual industrial apparatus:

$$(\rho_s)_2, \ d_2, \ u_2, \ D_2, \ (J_s)_2. \tag{3}$$

Our analysis of the system of nondimensional groups (1) obtained in [1] has shown that one of the most important specific features of a fluidized system has not been taken into account, namely, the existence of a developed large-scale motion of particles at gas velocities $u > u_t^*$, which largely determines the character of the transfer processes. This circumstance is associated with the use of the rate of gas filtration u taken as the characteristic velocity for both the gas and the particles. It is clear that this quantity does not characterize the velocity of the latter and, as a consequence, in the nondimensional groups (1) there is no characteristic of the above-mentioned large-scale motion of particles. Moreover, in Eq. (1) the most important scale_magnitude, i.e., the layer height, is absent, and some doubts arise about the validity of using the Fr_D and J_s numbers,

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which have no clear physical meaning. These remarks lead to the conclusion that it is necessary to substantially modernize system (1) and to obtain a new physically more adequate system of nondimensional groups which are free of the indicated drawbacks and reflect actual hydrodynamic processes in fluidized beds.

The Anderson–Jackson equations describing the hydrodynamics of the fluidized bed have the form [3]

$$\frac{\partial \varepsilon}{\partial t} = -\frac{\partial \varepsilon u_i}{\partial x_i}, \tag{4}$$

$$\frac{\partial (1-\varepsilon)}{\partial t} = -\frac{\partial ((1-\varepsilon) v_i)}{\partial x_i}, \tag{5}$$

$$\rho_f \varepsilon \left(\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} \right) = -\beta (u_i - v_i) - \frac{\partial p}{\partial x_i} + \mu_f \frac{\partial}{\partial x_k} \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right), \tag{6}$$

$$\rho_s (1-\varepsilon) \left(\frac{\partial v_i}{\partial t} + v_k \frac{\partial v_i}{\partial x_k} \right) = \beta (u_i - v_i) - \rho_s (1-\varepsilon) g \delta_{i3} - \frac{\partial p_s}{\partial x_i} +$$

$$+ \mu_s \frac{\partial}{\partial x_k} \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} \right), \tag{7}$$

and the boundary conditions

$$u_i (t, x, y, 0) = u \delta_{i3} \; ; \quad v_i (t, x, y, 0) = \frac{J_s}{\rho_s (1-\varepsilon)} \delta_{i3} \; ;$$

$$u_i (t, 0, y, z) = u_i (t, D, y, z) = u_i (t, x, 0, z) = u_i (t, x, D, z) = 0 \; ;$$

$$v_1 (t, 0, y, z) = v_1 (t, D, y, z) = v_2 (t, x, 0, z) = v_2 (t, x, D, z) = 0 \; ; \tag{8}$$

$$p (t, x, y, 0) = p_0 + \Delta p \, .$$

In writing system (4)-(8), we ignored the weight of the gas and the volume viscosities of the phases. The force of interaction between the gas and the particles is presented in the simplest form: $\beta(u_i - v_i)$, which in this case is admissible taking into account the objectives of the present work.

We introduce the nondimensional variables in the following manner:

$$x_i' = x_i / H^* \; ; \quad u_i' = u_i / u \; ; \quad v_i' = v_i / (u - u_t^*) \; ;$$

$$p' = p / \rho_f u^2 \; ; \quad p_s' = p_s / \rho_s (u - u_t^*)^2 \; ; \quad t' = ut / H^* \, . \tag{9}$$

It is significant to note that in the nondimensionalization use was made of two different characteristic velocities: u for the gas and $u - u_t^*$ for the particles. This circumstance, though quite insignificant at first glance, is of fundamental importance in the present case, because in the system of nondimensional parameters it allows one to take into account the noted specific feature of the fluidized system, i.e., the existence of the developed circulation motion of the particles. Since the latter are weighted by the gas flux, the entire excess power of the fan $\Delta p(u - u_t^*)$ is expended on producing and sustaining the large-scale circulation motion of the solid phase (as a rule, upward at the center of the apparatus and downward near its walls). Consequently, the quantity of the excess gas velocity $u - u_t^*$ characterizing the velocity of the rising motion of the particles is obviously the natural scale of their velocity.

With account for Eq. (9), we present the nondimensional form of system (4)-(8) as

$$\frac{\partial \varepsilon}{\partial t'} = -\frac{\partial \varepsilon u_i'}{\partial x_i'}, \tag{10}$$

$$\frac{\partial (1-\varepsilon)}{\partial t'} = -\frac{\partial ((1-\varepsilon) v_i')}{\partial x_i'} \frac{u - u_t^*}{u}, \tag{11}$$

$$\varepsilon \left(\frac{\partial u_i'}{\partial t'} + u_k' \frac{\partial u_i'}{\partial x_k'} \right) = -\frac{\beta H^*}{\rho_f u} \left(u_i' - v_i' \frac{u - u_t^*}{u} \right) - \frac{\partial p'}{\partial x_i'} + \frac{1}{\mathrm{Re}} \frac{d}{H^*} \frac{\partial}{\partial x_k'} \left(\frac{\partial u_i'}{\partial x_k'} + \frac{\partial u_k'}{\partial x_i'} \right), \tag{12}$$

$$(1-\varepsilon) \left(\frac{\partial v_i'}{\partial t'} \frac{u}{u - u_t^*} + v_k' \frac{\partial v_i'}{\partial x_k'} \right) = -\frac{(1-\varepsilon)\,\delta_{i3}}{\mathrm{Fr}_t^*} + \frac{\beta H^*}{\rho_s (u - u_t^*)} \left(u_i' \frac{u}{u - u_t^*} - v_i' \right) - \frac{\partial p_s'}{\partial x_i'} +$$

$$+ \frac{1}{\mathrm{Re}_s} \frac{d}{H^*} \frac{\partial}{\partial x_k'} \left(\frac{\partial v_i'}{\partial x_k'} + \frac{\partial v_k'}{\partial x_i'} \right), \tag{13}$$

and the boundary conditions as

$$u_i'(t', x', y', 0) = \delta_{i3}; \quad v_i'(t', x', y', 0) = \frac{\bar{J}_s^*}{1-\varepsilon} \delta_{i3};$$

$$u_i'(t', 0, y', z') = u_i'(t', D/H^*, y', z') = u_i'(t', x', 0, z') = u_i'(t', x', D/H^*, z') = 0;$$

$$v_1'(t', 0, y', z') = v_1'(t', D/H^*, y', z') = v_2'(t', x', 0, z') = v_2'(t', x', D/H^*, z') = 0;$$

$$p'(t', x', y', 0) = \frac{p_0}{\rho_f u^2} + \frac{\Delta p}{\rho_f u^2}. \tag{14}$$

Equations (10)-(14) contain the following nondimensional groups:

$$\frac{u - u_t^*}{u}, \quad \frac{\beta H^*}{\rho_f u}, \quad \mathrm{Re}, \quad \frac{d}{H^*}, \quad \mathrm{Fr}_t^*, \quad \frac{\beta H^*}{\rho_s (u - u_t^*)},$$

$$\mathrm{Re}_s, \quad \bar{J}_s^*, \quad \frac{D}{H^*}, \quad \frac{\Delta p}{\rho_f u^2}. \tag{15}$$

It is easy to show that not all of the quantities entering into Eq. (15) are independent. Representing the force of interphase interaction on the basis of the known Ergun formula [4], for $\beta H^* / \rho_f u$ we obtain

$$\frac{\beta H^*}{\rho_f u} = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{1}{\mathrm{Re}} \frac{H^*}{d} + 1.75 \frac{1-\varepsilon}{\varepsilon^3} \left| u_i' - v_i' \frac{u - u_t^*}{u} \right| \frac{H^*}{d} . \tag{16}$$

Equality (16) allows us to eliminate the complex $\frac{\beta H^*}{\rho_f u}$ from Eq. (15) as a parameter dependent on other parameters. For this reason, $\frac{\beta H^*}{\rho_s(u - u_t^*)} = \frac{\beta H^*}{\rho_f u} \frac{\rho_f}{\rho_s} \frac{u}{u - u_t^*}$ is also omitted. Taking into account the rather scanty knowledge of the rheological properties of disperse systems, it is worthwhile not to consider the number Re_s at all. Finally, using the equality $\mathrm{Ar} = \frac{\rho_s}{\rho_f} \mathrm{Re}^2 / \left(\mathrm{Fr}_t^* \left(\frac{u}{u - u_t^*} \right)^2 \frac{H^*}{d} \right)$, it is possible, instead of the simplex H^*/d, to substitute the Archimedes number into Eq. (15) and to eliminate the parameter $(u - u_t^*)/u = 1 - f(\mathrm{Ar})/\mathrm{Re}$*) as being the dependent one. The system of nondimensional groups that determine the similarity laws of transfer processes has the final form

$$\mathrm{Ar}\,,\ \mathrm{Re}\,,\ \mathrm{Fr}_t^*\,,\ \frac{\rho_s}{\rho_f}\,,\ \frac{D}{H_*}\,,\ \begin{Bmatrix} \bar{J}_s^* \\ \frac{\Delta p}{\rho_f u^2} \end{Bmatrix} . \tag{17}$$

The braces that contain two complexes are introduced for reflecting the features of operation of the apparatus with a circulating fluidized bed. As is known, two regimes of controlling the particle mass in a lifting standpipe are possible: a) the external circulation flux J_s, which in the present case is an independent parameter, is regulated, and $\Delta p/\rho_f u^2$ is eliminated from Eq. (17) as the dependent quantity; b) a pressure drop in the standpipe Δp is controlled and the complex $\bar{J}_s^*$ (as the dependent one) is eliminated from system (17).

With account for the equality $\Delta p = \rho_s(1 - \varepsilon_{mf}) g H_{mf}$ the parameter $\Delta p/\rho_f u^2$ can be presented in the form

$$\frac{\Delta p}{\rho_f u^2} = \frac{H_{mf}}{H^*} (1 - \varepsilon_{mf}) \frac{\rho_s}{\rho_f} \frac{1}{\mathrm{Fr}_t^*} \left(\frac{1 - \mathrm{Re}_t^* (\mathrm{Ar})}{\mathrm{Re}} \right)^2 , \tag{18}$$

which allows us to replace $\Delta p/\rho_f u^2$ by H_{mf}/H^* in Eq. (17):

$$\mathrm{Ar}\,,\ \mathrm{Re}\,,\ \mathrm{Fr}_t^*\,,\ \frac{\rho_s}{\rho_f}\,,\ \frac{D}{H^*}\,,\ \begin{Bmatrix} \bar{J}_s^* \\ \frac{H_{mf}}{H^*} \end{Bmatrix} . \tag{19}$$

System (19) was obtained previously in [5] on the heuristic basis using the π-theorem of the dimensional theory. In the case of a not flowing-through fluidized bed $J_s = 0$, $H_{mf}/H^* = 1$ and Eq. (19) is simplified:

$$\mathrm{Ar}\,,\ \mathrm{Re}\,,\ \mathrm{Fr}\,,\ \frac{\rho_s}{\rho_f}\,,\ \frac{D}{H_{mf}} . \tag{20}$$

From the conditions for the equality of the analogous criteria of system (19) (with $\bar{J}_s^*$) that are similar to Eq. (2)

*) Here the presence of the known relation $\mathrm{Re}_t^* = f(\mathrm{Ar})$ is used.

$$\mathrm{Ar}_1 = \mathrm{Ar}_2\ ;\ \ ...;\ (\bar{J}_s^*)_1 = (\bar{J}_s^*)_2\ , \tag{21}$$

it is easy to obtain the following relations for determining the parameters of the laboratory bench:

$$(\rho_s)_2 = (\rho_s)_1 \frac{(\rho_f)_2}{(\rho_f)_1}\ ,\quad d_2 = d_1 \left(\frac{(\nu_f)_2}{(\nu_f)_1}\right)^{2/3}\ ,\quad u_2 = u_1 \left(\frac{(\nu_f)_2}{(\nu_f)_1}\right)^{1/3}\ ,$$

$$D_2 = D_1 \left(\frac{(\nu_f)_2}{(\nu_f)_1}\right)^{2/3}\ ,\quad (J_s)_2 = (J_s)_1 \left(\frac{(\nu_f)_2}{(\nu_f)_1}\right)^{1/3} \frac{(\rho_f)_2}{(\rho_f)_1}\ ,\quad H_2^* = H_1^* \left(\frac{(\nu_f)_2}{(\nu_f)_1}\right)^{2/3}\ . \tag{22}$$

The first five relations in Eq. (22) coincide with those obtained in [1], which indicates the practical equivalence of Eqs. (1) and (19) in terms of finding general regularities of a scale transition in fluidized beds. Nevertheless, the system of criteria (19) obtained has considerable advantages over Eq. (1). First, system (19) is complete, since it gives six equations for determining all six parameters of the laboratory bench that models the industrial apparatus (including the equation for finding the bed height, which is absent in Eq. (2)). Second, the complexes Fr_t^* and $\bar{J}_s^*$ that are similar to Fr_D and J_s, in contrast to the latter have a clear physical meaning; Fr_t^* is the relation between the kinetic energy of the particles and their potential energy; $\bar{J}_s^*$ is the concentration of the particles in the upper portion of the bed. Both these parameters contain the quantity $u - u_t^*$, which characterizes the particle-rise velocity. In this connection, Fr_t^* and $\bar{J}_s^*$ should be considered as the important generalized characteristics of the large-scale convective motion of the particles; here Fr_t^* reflects the intensity of their internal circulation (the bed height is included in the composition of Fr_t^*) and $\bar{J}_s^*$ reflects the intensity of the external circulation of the particles in a flowing-through system. Since the convection of the particles exerts a determining influence on the transfer processes in fluidized systems, the use of the system of nondimensional complexes (19) containing Fr_t^* and $\bar{J}_s^*$ is very efficient for generalization of the experimental data on various transfer characteristics of disperse layers. As experience shows, numerous calculated correlations [5-7] obtained using Fr_t^* and $\bar{J}_s^*$ are simple and possess a large degree of universality. Undoubtedly, this fact is an important confirmation of the physical adequacy of system (19).

The similarity criteria established can also be used for other disperse layers, in which the weight of the particles is also compensated for by the force of friction on the gas, i.e., for vertical pneumatic transport and an incident blown-through layer. Since these layers are free of the internal circulation of the particles, system (17) is simplified: the number Fr_t^* is eliminated from this system and, in addition, the simplex D/H^* should be replaced by D/d:

$$\mathrm{Ar}\ ,\ \ \mathrm{Re}\ ,\ \ \frac{\rho_s}{\rho_f}\ ,\ \ \frac{D}{d}\ ,\ \ \bar{J}_s^*\ . \tag{23}$$

We note that in the present case the pressure drop is directly related to the magnitude of the particle flux

$$\frac{\Delta p}{\rho_s g H^*} = 1 - \varepsilon = \frac{\bar{J}_s^*}{1 + \bar{J}_s^*}\ . \tag{24}$$

This enables one not to introduce the complex $\Delta p/\rho_f u^2$ into Eq. (23). System (23) describes the similarity of the transfer processes in the indicated disperse layers (for the incident blown-through layer $\bar{J}_s^* = J_s/\rho_s(u_t^* - u)$)

*) The simplex D/d appears in the initial system of nondimensional groups (15) when D is taken as the characteristic dimension. In Eq. (15) the number Fr_t^* is replaced by the expression $\mathrm{Fr}_t^* = \frac{\rho_s}{\rho_f}\mathrm{Re}^2 \Big/ \left(\mathrm{Ar}\left(\frac{u}{u - u_t^*}\right)^2 \frac{H^*}{d}\right)$ with the replacement $H^* \to D$.

and makes it possible to determine the five parameters of Eq. (3) of the laboratory installation that models the industrial apparatus.

NOTATION

$\mathrm{Ar} = gd^3\rho_f(\rho_s - \rho_f)/\mu_f^2$, Archimedes number; d, particle diameter; D, width of the layer (riser); $\mathrm{Fr}_t^* = (u - u_t^*)^2/gH^*$, $\mathrm{Fr}_D = u^2/gD$, and $\mathrm{Fr} = (u - u_{mf})^2/gH_{mf}$, Froude numbers; g, free-fall acceleration; H, height of the layer (standpipe); $H^* = H_{mf}$, fluidized bed, $H^* = H$, circulating fluidized bed; H_{mf}, height of the layer with $u = u_{mf}$; J_s, specific mass flux of the particles; $\bar{J}_s^* = J_s/\rho_s(u - u_t^*)$ and $\bar{J}_s = J_s/\rho_s u$, nondimensional mass fluxes of the particles; p, pressure; p_0, pressure at the exit from the layer; Δp, pressure drop in the layer; $\mathrm{Re} = ud\rho_f/\mu_f$, $\mathrm{Re}_s = (u - u_t^*)d\rho_s/\mu_s$, and $\mathrm{Re}_t^* = u_t^* d\rho_f/\mu_f$, Reynolds numbers; t, time; u_i, gas velocity; u, rate of gas filtration; u_{mf}, velocity of the onset of fluidization; u_t, flotation velocity of a single particle; u_t^*, flotation velocity of the particles under constrainment conditions ($u_t^* \to u_{mf}$ for $\varepsilon \to \varepsilon_{mf}$, fluidized bed; $u_t^* \to u_t$ for $\varepsilon \to 1$, circulating fluidized bed); v_i, particle velocity; x_i, $i = 1, 2, 3$, horizontal (x, y) and vertical (z) coordinates; β, coefficient of resistance; δ_{ij}, Kronecker symbol; ε, porosity; μ, dynamic viscosity; ν, kinematic viscosity; ρ, density. Subscripts: f, gas; mf, onset of fluidization; s, particles; t, flotation conditions of the particles; 1, industrial apparatus; 2, cold laboratory bench that models the industrial apparatus.

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